Energy-Efficient Threshold Circuits Detecting Global Pattern in 1-Dimensional Arrays (New Trends in Theoretical Computer Science)

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Energy-Efficient Threshold Circuits Detecting Global Pattern in 1-Dimensional Arrays

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1 Introduction

Neurons communicate with each other by "firing" (i.e., emitting an electrical signal) for information processing, and a circuit consisting of neurons is often modelled by a combinatorial logic circuit, called a *threshold circuit*. Motivated by a biological fact that a neuron consumes substantially more energy to fire than not to fire [1], Uchizawa, Douglas and Maass proposed a complexity measure, called *energy complexity*, and initiate a study of threshold circuits with small energy complexity [2].

In this paper, we consider a Boolean function, called P_{LR}^n, which Legenstrin and Maass introduced to model a simple task for a pattern recognition on 1-dimensional array [3]. We investigate a relationship between the energy and size of a threshold circuit computing P_{LR}^n.

2 Preliminaries

A *threshold circuit* C is a combinatorial circuit of threshold gates. A threshold circuit C is expressed by a directed acyclic graph; let n be the number of input variables to C, then each node of in-degree 0 in C corresponds to one of the n input variables $x_1, x_2, \cdots, x_n$, and the other nodes correspond to threshold gates. We define *size* s of a threshold circuit C as the number of threshold gates in C. Let $g_1, g_2, \ldots, g_s$ be the gates in C. One may assume without loss of generality that $g_1, g_2, \ldots, g_s$ are topologically ordered with respects to the underlying graph of C. Let i be an integer such that $1 \leq i \leq s$. For each gate g_i, we denote by $w_{i,1}, w_{i,2}, \ldots, w_{i,l_i}$ the weights and by t_i the threshold of the gate g_i, respectively, where the weights and the threshold are real numbers and l_i is the fan-in of the gate g_i. Let $\boldsymbol{z}_i(\boldsymbol{x}) = (z_{i,1}(\boldsymbol{x}), z_{i,2}(\boldsymbol{x}), \cdots, z_{i,l_i}(\boldsymbol{x})) \in \{0,1\}^{l_i}$ be an input to g_i for a circuit input $\boldsymbol{x}$. The output $g_i(\boldsymbol{z}_i(\boldsymbol{x}))$ of g_i is defined as follows: $g_i(\boldsymbol{z}_i(\boldsymbol{x})) = 1$ if $\sum_{j=1}^{l_i} w_{i,j} z_{i,j}(\boldsymbol{x}) \geq t_i$; and $g_i(\boldsymbol{z}_i(\boldsymbol{x})) = 0$ otherwise. For every input $\boldsymbol{x} \in \{0,1\}^n$, the *output* $C(\boldsymbol{x})$ *of* C is denoted by $g_s(\boldsymbol{z}_s(\boldsymbol{x}))$. Let $f : \{0,1\}^n \to \{0,1\}$ be a Boolean function of n inputs. A threshold circuit C *computes* a Boolean function f if $C(\boldsymbol{x}) = f(\boldsymbol{x})$ for every input $\boldsymbol{x} \in \{0,1\}^n$. The *energy* e of a threshold circuit C is defined as the maximum number of gates outputting "1" in C, where the maximum is taken over all inputs to C.

For any positive integer n, we define P_{LR}^n as follows: For $\boldsymbol{x} = (x_1, x_2, \ldots, x_n) \in \{0,1\}^n$ and $\boldsymbol{y} = (y_1, y_2, \ldots, y_n) \in \{0,1\}^n$, $P_{LR}^n(\boldsymbol{x}, \boldsymbol{y}) = 1$ if there exists a pair of indices i and j such that $1 \leq i < j \leq n$, $x_i = y_j = 1$; and $P_{LR}^n(\boldsymbol{x}, \boldsymbol{y}) = 0$ otherwise.

3 Our result

We give a construction of energy-efficient threshold circuits computing P^n_{LR}. The following theorem gives an upper bound on the size of threshold circuits computing P^n_{LR} with energy e for any $e \geq 3$.

Theorem 1 *Let n be a positive integer. Then, there is a threshold circuit C computing P^n_{LR} such that C has energy $e \geq 3$ and size $s = O\left(e \cdot n^{2/(e-1)}\right)$.*

Thus, one can construct an energy-efficient circuit computing P^n_{LR} if it is allowable to use large size.

We also consider the extreme case where a threshold circuit has energy $e = 1$. We prove by construction that a linear number of gates is sufficient.

Theorem 2 *Let n be a positive integer. Then, there is a threshold circuit C computing P^n_{LR} such that C has energy $e = 1$ and size $s = \lceil n/2 \rceil$.*

The following theorem implies that the size of C given in Theorem 2 is optimal .

Theorem 3 *Let n be a positive integer. Let C be any threshold circuit computing P^n_{LR} with energy $e = 1$. Then, the size of C is at least $\lceil n/2 \rceil$.*

References

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